## Worksheet 1

A. Could there be:

1. Jointly inconsistent sentences, all of which are true?
2. Jointly inconsistent sentences, all of which are false?
3. A valid argument, all of whose premises are inconsistent?
4. An invalid argument, where the premises of the argument together with the conclusion are inconsistent?

In each case: if so, given an example; if not, explain why not.

We will take up the answers to these questions in class (so make sure you're there for that, and to pick up an answer sheet!). However, as there are going to be many questions of this kind throughout the course (and on the exam) it's important to learn the best method for answering them. l'll walk through the answers to A1 and A3 here in order to demonstrate.

In general, the best way to approach questions like these is to start with the precise technical definitions of the terms involved. So, beginning with question A1, we start by picking out the important terms (l've highlighted these), and defining them.

Could there be...
Jointly inconsistent sentences, all of which are true?
When defining the terms stay as close to the textbook definition as possible. In school, you're often told to "put things in your own words". Avoid that here! Commit the precise technical definitions of these terms to memory, and use those in your answers.

A set of sentences is jointly inconsistent just in case it is impossible for them all to be true together.

Now, the answer to A1 is NO. And since this is the case, there's no way we can demonstrate the answer by providing an example. So we need to explain the answer by appealing to the definition of the technical term(s) involved. Make sure you include the definition in the answer to provide.

> ANSWER: A set of sentences is jointly inconsistent just in case it is impossible for them all to be true together. So, by definition, there could not be a set of jointly inconsistent sentences, all of which are true together.

Now let's look at question A3.
Could there be...

## A valid argument, all of whose premises are inconsistent?

Start by defining the important terms:

## A valid argument is an argument in which it is impossible for all the premises to be true together and the conclusion to be false.

## A set of propositions is inconsistent if and only if it is impossible for all of the propositions to be true together.

Now, given these definitions, we consider the case described by the question. In this case, the answer is YES. So it's enough to just give a single example of the case described.

> ANSWER:
> P1. Snow is white.
> P2. Snow is not white.
> C. Dogs are mammals.

The reason why this is valid is slightly tricky, so l'll explain. At first glance, the argument I gave looks preposterous! How on earth could something like that be valid?!

It comes down to the technical definitions. The definition of validity requires that it be impossible for all the premises to be true together and, at the same time, for the conclusion to be false. Now, if the premises are inconsistent, then this is definitely impossible! Why? Because it's just impossible to do the first bit -- it's impossible for all the premises to be true together. And if it's impossible for the premises to be true together, then it's also impossible for them to be true together while the conclusion is false.

Think about it this way: Suppose you wanted a tin of biscuits with shortbreads and with bourbons. But I tell you there are no tins of biscuits with shortbreads in them. Well, there certainly aren't going to be any tins with shortbreads and bourbons then either.

So, 3 calls for a possibility where the premises are true together (shortbreads) and the conclusion is also false (bourbons). But the definition of validity tells us there are no possibilities where the premises are true together (no shortbreads!). So, there can't be any possibility where the premises are true together (shortbreads) and the conclusion is false (bourbons).

In other words, the situation described in 3 always satisfies the definition of validity (but never satisfies our desire for biscuits).
B. * Using the following symbolisation key:

B: Businesses will do something (about climate change)
C: Climate change will occur
D: The world is doomed
E: We cut carbon emissions by $60 \%$
G: Governments intervene (on climate change)
$S$ : Scientists are right (about climate change)
Symbolise the following sentences:

1. Businesses will not do anything and climate change will occur.
2. Scientists are right: in the event of climate change, the world is doomed.
3. If governments do not intervene, then businesses will do nothing and the world is doomed.
4. The world is doomed, since we will not cut carbon emissions by $60 \%$ whether or not governments intervene.
5. Only if businesses do something will we cut carbon emissions by $60 \%$.
6. If scientists are right, then we will avoid climate change if and only if we cut carbon emissions by $60 \%$.
7. Without government intervention, we are doomed unless scientists are wrong.
8. There are only two options, and they are exclusive: either we cut carbon emissions by $60 \%$, or the world is doomed.

Once again, l'll walk through how to approach these questions in general. Then I will explain the answers to the trickier ones.

The best way to approach a symbolisation question is to do it tiny step at a time. It can be daunting to try to just 'see' the answer, especially with the more complex sentences! Here's one method that you might find helpful. l'll use a new example to demonstrate.

## Ex. If we cut emissions by $\mathbf{6 0 \%}$, climate change will not occur.

## Step 1. Find the parts that match the symbolisation key.

Look at the natural language, and in some way highlight (underline or circle) the parts that match the symbolisation key.

If we cut emissions by $60 \%$, $\frac{\text { climate change will not occur }}{\text { E }}$.
At this point, I like to rewrite the sentence(s) in a kind of combination of natural language and symbols (propositional logic). So l'd rewrite the above like so:

$$
\text { If } E, \text { not } C .
$$

## Step 2: Find the connective words.

The natural language words you've got left are probably connective words. Often, these words (like 'if', 'or', 'and') make it fairly clear which logical connective we should be using in the formalisation. Sometimes, (like with 'since', 'but') it's a little less clear, but with practice (and coming to class!) you'll get the hang of these too. When I'm working things out on paper, I like to circle the connective words and write the connective I think I need above or below them. Here l'll highlight them.


## Step 3: Put it all together!

Replace the natural language with the symbols you've identified from the key. Et voilà!

$$
E \rightarrow \neg C
$$

Now, of course, some sentences are a lot more complicated than this. Sometimes it's not quite so easy to tell what operators you need. And sometimes, you need to jumble up the order of the natural language sentences to make the formalisation say the same thing!

Let's walk through a couple of the tougher questions.
B4. The world is doomed, since we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene.

One of the really tricky parts of this question lies in the use of 'since'. So, before tackling the question, let's better understand how 'since' works.

It's tempting to formalise 'since' as a conditional -- after all, in ordinary use, the word is usually used to suggest that one thing is, in some way, a consequence of the other.

Take the following example:

## Since it's raining, l'll take my umbrella.

Let ' $\mathbf{R}$ ' = It is raining
Let ' $\mathbf{U}$ ' = I'll take my umbrella

Here, it is tempting to formalise the sentences as follows:

$$
\mathbf{R} \rightarrow \mathbf{U}
$$

But this would be wrong! Or, at least, incomplete. If I tell you that if it's raining, I'll take my umbrella, then I haven't told you anything about what the weather is like right now. It may well not be raining right now.

But now, think about the original natural language sentence. If I were to say to you, "Since it's raining, I'll take my umbrella," would you think it might not be raining? Answer: no. By saying 'since' I imply that it is in fact raining. If it's still not that intuitive to you, imagine if I had said the same thing on a bright, sunny day. You'd immediately correct me: "But it's not raining" you might reply.

This is why the conditional alone is not enough. We need a formalisation of 'since it's raining, l'll take my umbrella' that is only true when $R$ is true, i.e. when it's raining. To do this, we can conjoin the conditional $R \rightarrow U$ with the proposition $R$. That is, we can formalise 'since it's raining, l'll take my umbrella' as follows:

$$
(R \rightarrow U) \wedge R
$$

You might notice that your answer sheet does not formalise 'since' in the same way as this. Instead, it uses a conjunction alone. In fact, these two strategies both work out. For instance, we might also formalise 'since it's raining, l'll take my umbrella' thus:

## $R \wedge U$

This is in fact equivalent to the previous formalisation. That is to say, the two propositions are true in exactly the same situations. If I tell you, "if it's raining, l'll take my umbrella," and then tell you, "and also, it's raining," what would you infer? You'd infer that l'll take my umbrella. In other words, you'd come to think that $\mathbf{R}$ and $\mathbf{U}$ are both true.

With all this in mind, let's return to the original question:

## The world is doomed, since we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene.

The suggestion here in the natural language seems to be that the world is doomed as a result of the fact that we will not cut carbon emissions by $60 \%$ whether or not governments intervene. Something like:

# (we will not cut carbon emissions by $60 \%$ whether or not governments intervene) $\rightarrow$ The world is doomed 

But, now we know that 'since' tells us something more than this. The use of 'since' here also tells us that it's true that we will not cut carbon emissions by $60 \%$ whether or not governments intervene. So to the conditional above, we'd have to add something; namely, we'd have to conjoin it with its antecedent such that:
[(we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene) $\rightarrow$ The world is doomed]
$\wedge$
[we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene]

Okay, that's the 'since' part out of the way. Now let's sort out how to formalise the bit still in natural language.

Given the symbolisation dictionary, 'the world is doomed' should be formalised as D. Let's stick that in first:
[(we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene) $\rightarrow$ D]
$\wedge$
(we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene)

We're not quite done, of course. How do we tackle the last bit in natural language? I.e.
(we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene)

The suggestion in the natural language is that, regardless of what governments do, we will not cut carbon emissions by $60 \%$. That is, if governments either intervene or they don't, then we will not cut carbon emissions by $60 \%$. So:

$$
(G \operatorname{v} \neg G) \rightarrow \neg E
$$

Now all that's left is to put the pieces together. We just formalised the proposition 'we will not cut carbon emissions by $60 \%$ whether or not governments intervene'. Let's plug that into this:

# [(we will not cut carbon emissions by $\mathbf{6 0 \%}$ whether or not governments intervene) $\rightarrow$ D] 

$\wedge$
[we will not cut carbon emissions by $60 \%$ whether or not governments intervene]

Plugging in $(\mathbf{G} \mathbf{v} \neg \mathbf{G}) \rightarrow \neg \mathbf{E}$, we get:

$$
(((\mathbf{G} v \neg \mathbf{G}) \rightarrow \neg E) \rightarrow \mathbf{D}) \wedge((\mathbf{G} \vee \neg \mathbf{G}) \rightarrow \neg E)
$$

Or, equivalently:

$$
((G \vee \neg G) \rightarrow \neg E) \wedge D
$$

There is another answer given on the sheet, however, using just conjunction. The reason why this works lies in the fact that $(\mathbf{G} \vee \neg \mathbf{G})$ is a tautology. That is to say, on every valuation (i.e. every line of the truth-table for this proposition) $(\mathrm{G} \vee \neg \mathrm{G})$ is true. Since for a conditional to be true, the consequent must be true whenever the antecedent is true, and since the antecedent $(\mathbf{G} \vee \neg \mathbf{G})$ is always true, it will always be true that $\neg \mathbf{E}$ (i.e the consequent of that conditional will always be true). In other words, it follows from the conditional ( $\mathbf{G} \vee \neg \mathbf{G}) \rightarrow \neg \mathbf{E}$ that $((\mathbf{G} \vee \neg \mathbf{G}) \wedge \neg \mathbf{E})$.

Thus, the following is equivalent to the preceding formalisation of 4 .

$$
((G \vee \neg G) \wedge \neg E) \wedge D
$$

B5. Only if business do something will we cut emission by $\mathbf{6 0 \%}$.
Here, the tricky bit is 'only if'. So before we formalise this sentence, let's get to grips with how 'only if' works. To do this, let's think about the following example:

## Only if it's cold out will it snow.

The 'if' bit suggests that we are dealing with a kind of conditional. The question is, which part of the sentence should come before the arrow, and which part after it? Before we formalise anything, let's think about what the natural language sentence is trying to tell us. If I told you 'only if it's cold out will it snow', and then it starts snowing, what do you know about the temperature? Well, it's got to be cold! Why? Because I told you that the only situation in which it'll snow is if it's cold outside. In other words, whenever it's true that it's snowing, it's also true that it's cold.

Now, suppose we formalised 'only if it's cold out will it snow' this way:

$$
\mathbf{C} \rightarrow \mathbf{S} \quad x
$$

This would be wrong. This formalisation says that whenever it's cold, then it snows. But that's completely different from saying that it only snows when it's cold (which is what the original sentence says). After all, there are plenty of cold days that aren't snowy! The problem with this formalisation is that the arrow is going in the wrong direction. What we should have done is this:

$$
\mathbf{S} \rightarrow \mathbf{C}
$$

So, in general then, ' $\mathbf{X}$ only if $\mathbf{Y}$ ', or equivalently 'only if $\mathbf{Y}, \mathbf{X}$ ' should be formalised ' $\mathbf{X} \rightarrow \mathbf{Y}$ '.
With this in hand, let's return to the original question...

Only if business do something will we cut emission by $\mathbf{6 0 \%}$.
We can apply our general method...
Step 1. Find the parts that match the symbolisation key.
Only if business do something will we cut emission by $\mathbf{6 0 \%}$.
$B \quad E$
So from this we get:
Only if B, E
Step 2. Find the connective words.
Only if B, E

## Step 3. Put it all together!

Above, we worked out the way 'only if' conditionals work. Based on that model, we know that the natural language is telling us that the only situation in which we cut emissions is one in which businesses do something. So the conditional should look like:

$$
E \rightarrow B
$$

## B7. Without government intervention, we're doomed unless scientists are wrong.

The key to doing this formalisation lies in understanding 'unless'. So, as above, we'll figure out how 'unless' works before tackling the question itself.

In general, 'unless' statements of the form ' $\mathbf{X}$ unless $\mathbf{Y}$ ' tell you that the only situation in which $\mathbf{X}$ fails is if $\mathbf{Y}$ holds. So take 'You'll fail the test, unless you study.' (Note: we can switch the order of the clauses in natural language without changing the meaning of the sentence: 'Unless you study, you'll fail the test.') This statement tells us that the only situation in which you won't fail is if you study.

Now, if there is only one situation (namely you studying) in which you won't fail, then one of two things can happen: either you'll fail or you won't. And if you don't fail, then that's because you studied. So, either you'll fail, or you'll study. Hence, if we let 'F' symbolise 'you will fail the test', and let ' $\mathbf{S}$ ' symbolise 'you will study', 'You'll fail the test unless you study' can be symbolised:

## FvS

Let's try another example. Consider the sentence 'I won't go unless you come with me.' We can imagine someone saying this as a kind of ultimatum. Here, the person is saying that the only situation in which they'll go is if you go with them. Indeed, we can even imagine someone huffily putting their foot down and saying 'Either you're coming with me, or l'm not going.'

Letting ' $\mathbf{Y}$ ' symbolise 'you're coming with me' and ' $\mathbf{G}$ ' symbolise 'l'm going', 'I won't go unless you come with me' would be formalised as:

$$
\neg G \operatorname{v}
$$

We can also think of this in conditional terms. The statement 'l'm not going unless you come with me' suggests that there is some relation between your coming with me and my going. Hence: 'If you don't come with me, then I won't go.' Or, issued as an ultimatum again: 'If I'm going, then you're coming with me!'. Respectively, these are formalised as follows:

$$
\neg \mathbf{Y} \rightarrow \neg \mathbf{G}
$$

or

$$
\mathbf{G} \rightarrow \mathbf{Y}
$$

These conditionals and the disjunction above them are all equivalent to one another. Once again, you can use truth tables to prove this.

With all this in mind, let's return to the original question:

Without government intervention, we're doomed unless scientists are wrong.

Let's deal with the 'unless' bit first. That is, this part:

## we're doomed unless scientists are wrong

As above, this suggests that the only situation in which we're not doomed is that in which scientists are wrong. In other words, if we're not doomed, then scientists are wrong. Put differently again, we are either doomed, or scientists are wrong. Hence:

$$
\begin{gathered}
\mathrm{D} v \boldsymbol{\sim} \mathrm{~S} \\
\mathrm{OR} \\
\neg \mathrm{D} \rightarrow \boldsymbol{\sim}
\end{gathered}
$$

OR

$$
\mathbf{S} \rightarrow \mathbf{D}
$$

For the rest of the solution, l'll use the first formalisation of the 'unless' claim (i.e. ' $\mathbf{D} v \neg \mathbf{S}$ ). However, all of the formalisations work equally well.

Okay, let's stick this back into the original sentence. Substituting the 'unless' claim for the formalisation we've come up with, we're left with:

## Without government intervention, ( $\mathrm{D} v \mathrm{a}$ )

Now we need to sort out how to formalise the rest of the sentence. The use of the word 'without' here suggests that there is a relation between government intervention and ( $\mathbf{D} v$ $\neg \mathbf{S}$ ). Notice, too, that 'without' not only tells us about the connection between government intervention and ( $\mathbf{D} \vee \neg \mathbf{S}$ ), it also has a kind of built in negation. (Consider how the sentence

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$$
\text { If }\urcorner G \text {, then ( } D \text { v } \neg \text { S) }
$$

Which we can then formalise as...

$$
\neg G \rightarrow(D v \neg S)
$$

