1A Formal Methods
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Worksheet 3

A. Using the following symbolisation key:

domain: people
a: Ayn
m: Milton
Ixy: _____$_x$ influenced _____$_y$
Dxy: _____$_x$ dislikes _____$_y$
Lx: _____$_x$ is a libertarian

symbolise the following English sentences in FOL as best you can:

In this worksheet, the task was the symbolise a series of natural language sentences in *first order logic*. In what follows, I'll walk through my procedure for answering each of these questions. The hope is that it will help to guide you through future formalisation exercises. I'll go through the questions one at a time.

1. Since Ayn is a libertarian, Milton must not dislike her.

The first thing to do is to identify words that are logical connectives, or clues to logical operators. In this case, these are the words in red below:

Since Ayn is a libertarian, Milton must not dislike her.

Let's ignore the operators for now. The next step is to find constituent parts for which we have symbolisations in our dictionary. These are the following bracketed sections (distinct sections are numbered):

Since [Ayn is a libertarian]$_1$, [Milton must not dislike her]$_2$.

We'll symbolise these first, leaving the rest of the natural language as it is. I'll start with bracket #1:

Ayn is a libertarian

According to our symbolisation dictionary entries for 'Ayn' and 'is a libertarian', this should be symbolised:

La

The contents of bracket #2 read as follows:

Milton must not dislike her

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We know that the ‘her’ here is meant to refer to Ayn, so this says:

Milton must not dislike Ayn

But, it looks like we don’t have a way of symbolising ‘*must* not like Ayn’. So what do we do? Well, what’s expressed is mostly just a claim about Milton’s liking or disliking Ayn. Given the things we are able to say (according to our dictionary) it can be paraphrased:

Milton does not dislike Ayn

Have we lost something in the paraphrase? Perhaps. But this is the closest our first order language will be able to say, given the tools we’ve got. This represents a certain poverty in the language we’ve got. To express things like ‘must’, ‘could not have been otherwise’, ‘not possibly not’, we need a richer language than FOL (in particular, we need *modal* logic). But this is all just an aside. What matters for our purposes is the practice of paraphrasing. As we go through the rest of the questions, this tool will be increasingly useful.

Okay, let’s symbolise our paraphrase, then. We’ve got a ‘not’ in there, which is a logical operator word. I’ll work around it for now. So,

not Milton does dislike Ayn

should be symbolised:

not ***Dma***

Putting the symbolisations back in their respective brackets, we can go from what we started with:

Since **[Ayn is a libertarian]$_1$, [Milton must not dislike her]$_2$**

to the following:

Since **[*La*]$_1$, [not *Dma*]$_2$**

RIght. Now let’s deal with the logical operators! Right now we have:

Since *La*, not *Dma*

The ‘not’ is the most straightforward to deal with. Symbolising the ‘not’ we get the following:

Since *La*, ¬*Dma*

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And now we're just left with 'since'. I won't go through another explanation of how 'since' works here. You can refer back to that lesson sheet for a refresher. The thing to notice here is that, the task we have in symbolising

Since *La*, ¬*Dma*

is just the same as the task we have we symbolising anything that looks like this:

Since *P*, Q

The second should look like

P* ∧ *Q

And, for exactly the same reason, the first should look like:

La* ∧ ¬*Dma

And that's our answer to question 1!

2. Only if Ayn influenced him is Milton a libertarian.

We'll use the same procedure here as we did above.

STEP 1: Identify indicator words for logical operators.

Only if Ayn influenced him is Milton a libertarian.

STEP 2: Identify the parts of the sentence that correspond to expressions in our symbolisation dictionary.

Only if [Ayn influenced him]$_1$ [is Milton a libertarian]$_2$.

STEP 3: Working with these parts one at a time, paraphrase into an expression found in the symbolisation dictionary.

So,

(1) Ayn influenced him

becomes,

(1) Ayn influenced Milton

And,

(2) is Milton a libertarian

becomes,

(2) Milton is a libertarian

STEP 4: Symbolise the paraphrased pieces.

So,

(1) Ayn influenced Milton

becomes,

(1) Iam

And,

(2) Milton is a libertarian

becomes,

(2) Lm

STEP 5: Put the symbolised parts back into the appropriate place. (We'll use the numbers as a guide. You can do this just by writing underneath the original sentence, or something like that.)

Only if $[Iam]_1$ $[Lm]_2$

STEP 6: Symbolise the logical operators.

So,

Only if *Iam, Lm*

becomes the following, given the rules for 'only if':

$$\boldsymbol{Lm \rightarrow Iam}$$

And that's your answer.

Concerning 'only if', here's why the arrow has to go in this direction. 'Only if' statements are statements where we are told that there's only ONE situation in which something will happen. Consider 'Only if it rains will we cancel the picnic.' This tells you that we're definitely going to have a picnic *unless* it rains. That is, there is only one situation in which we'll cancel the picnic--namely, in the event of rain.

Recall the truth-table for '→'.

A	B	A → B
T	T	T
T	F	F
F	T	T
F	F	T

The arrow comes out true whenever the antecedent is false! So if we symbolised

Only if it rains will we cancel the picnic

as follows:

Rain → Cancel ❌

it leaves open the possibility that **Cancel** is true while **Rain** is false. That is, it allows for the possibility that we cancel the picnic when it's NOT raining! But that's precisely what we said wasn't possible earlier. We said there was only ONE situation in which we'd cancel--in the event of rain. In other words, if we cancel, then it must be raining. Hence:

Cancel → Rain ✔

Similarly, in the question from Worksheet 3, there's only ONE situation in which Milton is a libertarian, namely that in which Ayn influenced him. So, if Milton's a libertarian, Ayn influenced him. Hence:

$$\boldsymbol{Lm \rightarrow Iam}$$

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3. Unless Ayn is a libertarian whom Milton doesn't dislike, Milton is a libertarian.

This one's a bit more tricky, but the procedure remains the same.

STEP 1: Identify indicator words for logical operators.

Unless Ayn is a libertarian whom Milton doesn't dislike, Milton is a libertarian.

But why have I picked out 'whom'? To see why, consider this clause of the sentence:

Ayn is a libertarian whom MIlton doesn't dislike

Here, the 'whom' tells me that what comes before it is true, *and* so is what comes after it. I could paraphrase this clause as follows:

Ayn is a libertarian AND Milton doesn't dislike her

Now we can see that 'whom' was doing a kind of double duty. It was acting a bit like a conjunction, but one that also told me what exactly Milton was disliking. So many shortcuts in natural language! I'm going to use the paraphrase for the rest of the solution. So, we're now working with the following:

Unless Ayn is a libertarian and Milton doesn't dislike her, Milton is a libertarian.

STEP 2: Identify the parts of the sentence that correspond to expressions in our symbolisation dictionary.

Unless **[Ayn is a libertarian]$_1$** and **[Milton** doesn't **dislike her]$_2$, [Milton is a libertarian]$_3$.**

STEP 3: Paraphrase these into expressions found in the symbolisation dictionary.

[Ayn is a libertarian]$_1$

[not **Milton dislikes Ayn]$_2$**

[Milton is a libertarian]$_3$

STEP 4: Symbolise the paraphrased pieces.

[*La*]$_1$

[not **Dma]$_2$**

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[Lm]$_3$

STEP 5: Put the symbolised bits back.

Unless **[*La*]**$_1$ and **[**not ***Dma*]**$_2$**, [*Lm*]**$_3$**.**

STEP 6: Symbolise the operators.

Unless *La* and not *Dma*, *Lm*.

Working in order from right to left:

Unless *La* and ¬*Dma*, *Lm*.

then…

Unless *La* ∧ ¬*Dma*, *Lm*.

Now the slightly trickier bit. The comma is doing VERY important work here. It's basically our natural language version of a set of brackets. Here, the comma tells us the scope of the conjunction:

Unless (*La* ∧ ¬*Dma*) *Lm*.

Finally, the 'unless':

(*La* ∧ ¬*Dma*) v *Lm*

Or, alternatively:

¬*Lm* → (La ∧ ¬*Dma*)

(See the Worksheet 1 handout for an explanation of 'unless').

4. No libertarian dislikes either Ayn or Milton.

Now we're adding quantifiers into the mix. This means is that we have extra operator indicator words to look for in Step 1. Also, or paraphrasing and symbolising will go a bit differently, as you'll see.

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STEP 1: Identify indicator words for logical operators.

No libertarian dislikes either Ayn or Milton.

The next step is slightly different to what we were doing before. In a way, we'll be doing a combination of Steps 2 and 3 here (Step 3 was the paraphrasing step). I can see by the phrase 'no libertarian' that I'll be quantifying over things in this sentence. So, it will be easier if I paraphrase the entire sentence first and *then* find the bits that can be found in the symbolisation dictionary. So, in a way, I'm doing Step 3 first, and then Step 2.

STEP 3: Paraphrase.

The first thing to address is how to approach 'No _______'. Take the sentence we have above. It's basically of the form 'No ________ is such that __________'. Or,

No <u>libertarian</u> is such that <u>it dislikes either Ayn or Milton</u>.

Now we can deal with the 'No' directly. 'No libertarians' tells us that we are talking about *all* of the libertarians, and saying that something (the stuff in the blank after the 'such that') is *not* the case for them. So, we can paraphrase this:

For all libertarians it is not the case that they dislike either Ayn or Milton.

Now we can go one step further, paraphrasing this into something that is something like a logic-English hybrid. 'Log-lish' if you like.

Right now, the sentence tells us that *something* is true of ALL libertarians. In particular, the thing that is true of all libertarians is that "it is not the case that they dislike either Ayn or Milton". I'm just going to call that '**P**' for now. **P** is our placeholder for something that's true of all libertarians.

Now, if something is true of ALL libertarians, then take anything you like, IF it's a libertarian, THEN **P** will be true of it.

This is why universally quantified statements are often symbolised as conditionals. Suppose I told you that all elephants are grey. In log-lish, I've told you, for anything you like, if that thing is an elephant, then that thing is grey. Or, "for any x, if x is an elephant, then x is grey."

Similarly for the sentence about libertarians. Hence:

For all x, if x is a libertarian, then P.

where **P** was a placeholder. So, putting the rest back in:

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For all x, if x is a libertarian, then it is not the case that x dislikes either Ayn or Milton.

Which can be further paraphrased:

For all x, if x is a libertarian,
then it is not the case that x dislikes Ayn or x dislikes Milton.

The trouble is… now the scope of 'it is not the case that' is unclear! I'll put a pair of brackets in just to keep things clear in our own heads.

For all x, if x is a libertarian,
then it is not the case that (x dislikes Ayn or x dislikes Milton).

Okay, *now* let's do Step 2.

STEP 2: Identify the parts of the sentence that correspond to expressions in our symbolisation dictionary.

For all x, if [x is a libertarian]$_1$,
then it is not the case that ([x dislikes Ayn]$_2$ or [x dislikes Milton]$_3$).

STEPS 4 and 5: Symbolise the bracketed bits and put them in the right spot in the sentence

For all x, if [*Lx*]$_1$,
then it is not the case that ([*Dxa*]$_2$ or [*Dxm*]$_3$).

STEP 6: Symbolise the logical operators (making sure to mind the commas!).

The first bit is the quantifier. We can see that it has to range over the whole sentence because there are x's all the way through, and they all need to be within the quantifier's scope. So,

For all x, if *Lx*,
then it is not the case that (*Dxa* or *Dxm*).

becomes:

∀x(if *Lx*
then, it is not the case that (*Dxa* or *Dxm*))

Now the 'if …, then':

∀x(*Lx* → it is not the case that (*Dxa* or *Dxm*))

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Now ‘or’:

∀x(*Lx* → it is not the case that (*Dxa* v *Dxm*))

And finally, the ‘it is not the case that’:

∀x(*Lx* → ¬(*Dxa* v *Dxm*))

That’s all you need to know. But if you’re wondering, you can also symbolise claims like this one using the existential quantifier. To say

No Ps are Qs

is also equivalent to saying

There is nothing that is both a P and a Q

Or, in log-lish:

There does not exist an x such that x is P and x is Q

Which is symbolised:

¬∃x(Px ∧ Qx)

So, the sentence from Question 4 could also be paraphrased:

There doesn’t exist an x such that
x is a libertarian and x dislikes Ayn or x dislikes Milton

Formally:

¬∃x(*Lx* ∧ (*Dxa* v *Dxm*))

5. Some libertarians - including some who are influence by her - dislike Ayn.

FIND OPERATOR WORDS:

Some libertarians - including some who are influence by her - dislike Ayn

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This one's a deceptive one because we can't even pick out all of the logical operators until we've paraphrased it! So let's start with the paraphrasing.

PARAPHRASE:

The trick to paraphrasing this one lies in understanding 'some'.

Rule of thumb: whenever you see 'some', think: "there exists a".

You might be thinking, "But wait! 'Libertarians' was plural in the natural language. Doesn't that mean there's more than one? Not just <u>*a*</u> something or other?"

You'd think this, but in logic, that's not how 'some' works. Here 'some' means "there exists at least one". (For a bit of logic-related silliness, here's a sketch from Sesame Street wherein Grover inadvertently demonstrates his command of logic! http://youtu.be/UkmuVKvU0cY (Clearly the customer is not always right...))

To make our life easier, let's move the parenthetical remark to the end of the sentence. Hence:

Some libertarians dislike Ayn, including some who are influenced by her.

Okay, so we can see that the first bit tells us that there exists at least one libertarian that dislikes Ayn.

But what do we do with the second part? It is telling us that there's some kind of overlap between the libertarians that dislike Ayn and the libertarians that were influenced by her. How much overlap? Well, it says the group of libertarians that dislike Ayn (which potentially only has one member) *includes* "some"--at least one!--member that was also influenced by her. So, all together, the sentence can be paraphrased:

There exists a libertarian that dislikes Ayn and was influenced by her

In log-lish:

**There exists an x such that x is a libertarian, and x dislikes Ayn,
and x was influenced by Ayn**

Before moving on, it's important to note what the natural language sentence does *not* tell us. It <u>doesn't</u> tell us that there are any libertarians that *were not* influenced by Ayn and also dislike her.

It is tempting to read the original natural language sentence and think that it is telling us something like "There's a bunch of libertarians that dislike Ayn: some of them were influenced

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by her and some of them weren't." But this would be wrong. For all that the natural language sentence has said, *it might be the case that there is only one libertarian and that that person dislikes Ayn and was influenced by her*.

Okay, with that clear, let's return to symbolising the sentence. Now that we have a clear paraphrase, it's much easier to pick out all of the logical operator words:

There exists an x such that x is a libertarian, and x dislikes Ayn, and x was influenced by Ayn

We can also pick out the sections that correspond to the dictionary:

There exists an x such that [x is a libertarian], and [x dislikes Ayn], and [x was influenced by Ayn]

Those bracketed sections can be symbolised now.

There exists an x such that [*Lx*], and [*Dxa*], and [*Ixa*]

And finally, we can symbolise the logical operators.

$$\exists x(Lx \wedge Dxa \wedge Ixa)$$

And we're done!

6. Some libertarian influenced by Ayn dislikes exactly the same people as Milton dislikes.

As before, the best way to handle this sentence is to take it in parts. Here, the 'some' tells us that there *exists* a particular person. So, we can paraphrase as follows:

There exists an x that is a libertarian influenced by Ayn,
and
x dislikes exactly the same people as Milton dislikes

What do we learn when we hear that x "is a libertarian influenced by Ayn"? Well, we learn that x is a libertarian AND x is influenced by Ayn. So, to add that to our paraphrase:

There exists an x such that, x is a libertarian and x is influenced by Ayn
and
x dislikes exactly the same people as Milton dislikes

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Now we can pick out operator words:

There exists an x such that, x is a libertarian and x is influenced by Ayn
and
x dislikes exactly the same people as Milton dislikes

But what to do with the final conjunct…? “x dislikes exactly the same people as Milton dislikes.” The first clue is that the operator phrase is ‘exactly the same as’

There exists an x such that, x is a libertarian and x is influenced by Ayn
and
x dislikes exactly the same people as Milton dislikes

Okay, let’s not worry about the final conjunct for a moment. Instead, let’s get the earlier stuff out of the way.

There exists an x such that, x is a libertarian and x is influenced by Ayn
and
x dislikes exactly the same people as Milton dislikes

In the first line, we can pick out the bits for which we have expressions in our dictionary:

There exists an x such that, [x is a libertarian] and [x is influenced by Ayn]
and
x dislikes exactly the same people as Milton dislikes

And we can formalise them.

There exists an x such that, [*Lx*] and [*Iax*]
and
x dislikes exactly the same people as Milton dislikes

Let’s formalise the operators too.

∃x(*Lx* ∧ *Iax* ∧
x dislikes exactly the same people as Milton dislikes)

Okay, so we’ve formalised most of the sentence. We know that there’s some x, and a series of things are true of that x. All that’s left is to formalise the last thing that’s true of x -- namely, that they dislike exactly the same people Milton dislikes.

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∃x(*Lx* ∧ *Iax* ∧
x dislikes exactly the same people as Milton dislikes)

To understand ‘exactly the same as’, I’m going to use a different example. Once we’ve understood that, then we’ll return to the people x and Milton dislike.

Consider the following sentence:

Andy has exactly the same friends as Beth.

And here’s a symbolisation dictionary:

a: John
b: Beth
Fxy: $\underline{\hspace{3em}}_x$ is a friend of $\underline{\hspace{3em}}_y$

The first thing to notice is that this sentence doesn’t actually tell us how many friends either Beth or Andy has! In other words, we learn that

For any friends Andy has, they are exactly the same friends as Beth has.

This paraphrase gives us another set of operator words:

For any friends Andy has, they are exactly the same friends as Beth has.

And ‘any’ is a good indication that we need the *universal* quantifier. So

For all friends Andy has, they are exactly the same friends as Beth has.

For all *what*, though? For all friends of Andy. I.e.

For all x such that x is a friend of Andy, they are exactly the same friends as Beth has.

And we know that the x’s (if any) that are friends of Andy are exactly the same x’s that are friends of Beth. So,

For all x such that x is a friend of Andy, exactly the same x are friends of Beth.

Using our dictionary, we can symbolise ‘x is a friend of Andy’ and ‘x is a friend of Beth’:

∀x(*Fxa* exactly the same as *Fxb*)

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But what's the operator that should go in between *Fxa* and *Fxb*? Well, if we're talking about all the friends of Andy, then we are saying that, for all x, *if* x is a friend of Andy, then something is the case. So it's tempting to do this:

$$\forall x(Fxa \rightarrow Fxb)$$ ❌

But this would be wrong. To see why, consider the truth-table for '→' again. The conditional is going to turn out to be true even when the antecedent is false. So, imagine Colin was a friend of Beth but not a friend of Andy. There would be something for which *Fxb* is true, but *Fxa* is false. Namely, Colin. But, *the conditional is still true*! Despite Colin, it's still true that, for all things that are friends of Andy, they are friends of Beth. The problem is, with the conditional only going one way *the opposite doesn't hold*!

In order for Beth and Andy to have *exactly the same* friends, both conditionals have to hold. It must be true that, for all things that are friends of Andy, they're friends of Beth. AND, it must be true that for all things that are friends of Beth, they're friends of Andy. Hence:

$$\forall x(Fxa \longleftrightarrow Fxb)$$ ✔

Okay, so now we know that 'exactly the same as' should be formalised with a biconditional. Let's return to our original question.

∃x(*Lx* ∧ *Iax* ∧
x dislikes exactly the same people as Milton dislikes)

We want to say that x dislikes exactly the same people as Milton dislikes. Or, in other words,

∃x(*Lx* ∧ *Iax* ∧
For all y such that, x dislikes y, Milton dislikes exactly the same y)

Formally:

$$\exists x(Lx \wedge Iax \wedge \forall y(Dxy \longleftrightarrow Dmy))$$

7. Nobody dislikes anybody who dislikes every libertarian.

As with Question 4, this is a nonexistence claim, and so there are two different way we might symbolise it. I'll run through both.

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The best way to approach this question is by trying to symbolise ‘nobody dislikes anybody’ first. The full sentence tells us that nobody dislikes anybody *of some particular description*. So let’s not worry about the description just yet. Let’s start by working with

Nobody dislikes anybody

Here, we want to talk about *everyone* and say of them that there isn’t *anyone* they dislike. So,

For all x, there isn’t a y that x dislikes

Or,

For all x and for all y, it’s not the case that x dislikes y

Hence:

$$\forall x \forall y \neg Dxy$$

Okay. Now let’s return to the full sentence. We’ve just formalised nobody dislikes *anybody*, but the original sentence tells us something more precise than this. It says that nobody dislikes anybody *who dislikes every libertarian*. In other words,

For all x and for all y <u>that dislike every libertarian</u>, it’s not the case that x dislikes y

We need to restrict the kind of *y* that the x’s don’t dislike. So, we need to pick out the y’s that dislike every libertarian.

For y to dislike every libertarian, it has to be the case that,

if z is a libertarian, then y dislikes z

For which z? All of them. So,

For all z, if z is a libertarian, then y dislikes z

Formally:

$$\forall z(Lz \rightarrow Dyz)$$

Great. Now we’ve said the y’s dislike every libertarian. Let’s try to put this together with what we had before now.

Earlier, we formalised ‘nobody dislikes anybody’ as follows:

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$$\forall x \forall y \neg Dxy$$

But we didn't want all the x's not to dislike ALL the y's. We wanted them not to dislike y's *provided the y's dislike every libertarian*. This is a condition on the x's not disliking y's. So,

∀x∀y(IF (y dislikes every libertarian) THEN ¬*Dxy*)

And we just finished formalising 'y dislikes every libertarian', so we can put that in the brackets:

∀x∀y(IF (∀z(*Lz* → *Dyz*)) THEN ¬*Dxy*)

And lastly, formalise the 'if… then':

$$\forall x \forall y((\forall z(Lz \rightarrow Dyz)) \rightarrow \neg Dxy)$$

That's the formalisation using the universal quantifier. Now let's try it again with the existential.

As before, let's start with 'nobody dislikes anybody'. Paraphrased for the existential quantifier we have:

There doesn't exist an x such that x dislikes anybody

Okay, how do we paraphrase 'anybody' here? You might think that it's just the same as the way we did it before. So, you might be tempted to do this:

There doesn't exist an x such that, for all y, x dislikes y ❌

But this would be wrong! This says that there doesn't exist an x that dislikes ALL the y's. I.e. we've just said 'nobody dislikes *everybody*.' And that's not what we wanted! We want:

There doesn't exist an x such that x dislikes *somebody* ✔

In other words:

There doesn't exist an x such that there is *some* y that x dislikes

Formally:

$$\neg \exists x \exists y Dxy$$

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Okay, now we need to add the qualification to y. We don't want to say that there's no person that dislikes some y *period*. We want to say that there's no person that dislikes some y *that dislikes every libertarian.*

Of course, we've formalised 'y dislikes every libertarian' already. That looked like this:

$$\forall z(Lz \rightarrow Dyz)$$

So now we need to put this together with the formalisation of 'nobody dislikes anybody'. We want to say that there doesn't exist an x who dislikes y, where y dislikes every libertarian. Hence:

$$\neg \exists x \exists y(Dxy \land \forall z(Lz \rightarrow Dyz))$$

8. Milton dislikes anyone who is neither a libertarian nor influenced by one.

Let's use the same strategy here as we did before. Let's symbolise 'Milton dislikes anyone' first. And then we can add in the restriction on the people that Milton dislikes.

First: 'Milton dislikes anyone'. This is paraphrased:

For all x, Milton dislikes x

Hence:

$$\forall x Dmx$$

Now we need to say of x that 'x is neither a libertarian nor influenced by one.' This can be paraphrased:

it is not the case that (x is a libertarian OR x is influenced by a libertarian)

Some of these are quite straightforward to symbolise. So, let's take care of these first:

¬(Lx V x is influenced by a libertarian)

The question now is, how do we formalise 'x is influenced by a libertarian'? To do this, let's use the same strategy as we used above--we'll paraphrase it first.

x is influenced by a libertarian

This tells us that x is influenced by *some* libertarian. Whenever you see ‘a’ as in ‘a libertarian’ thing “some”. And, as we said before, “some” we take to mean “there exists”. So, we can further paraphrase the above as:

there exists a libertarian and x is influenced by them

Or, more precisely,

there exists a y such that y is a libertarian and x is influenced by y

Now let’s formalise this.

there exists a y such that (*Ly* and *Iyx*)

which becomes:

$$\exists y(Ly \land Iyx)$$

Okay, let’s put that back where ‘x is influenced by a libertarian’ was. So,

¬(Lx ∨ x is influenced by a libertarian)

becomes,

$$\neg(Lx \lor x\ \exists y(Ly \land Iyx))$$

This (the formal proposition immediately above) is the formalisation of ‘it is not the case that x is a libertarian or was influenced by one’.

Now we need to return to the original sentence. The original read: “Milton dislikes anyone who is neither a libertarian nor influenced by one.” We symbolised ‘MIlton dislikes anyone’ as follows:

$$\forall x Dmx$$

And we just finished symbolising the description of the x’s that Milton dislikes--namely, those that are neither libertarians nor were influenced by one. So, we want to say of all x’s meeting *that* description that Milton dislikes them. In other words IF an x meets that description, then Milton dislikes that x. Hence:

$$\forall x[\neg(Lx \lor x\ \exists y(Ly \land Iyx)) \rightarrow Dmx)$$